PHYS 2020: College Physics
Manic Monday #6

"I sometimes ask myself how it came about that I was the one to develop the theory of relativity. The reason, I think, is that a normal adult never stops to think about problems of space and time. These are things which he has thought about as a child. But my intellectual development was retarded, as a result of which I began to wonder about space and time only when I had already grown up." ∼ Albert Einstein

Special Relativity: In all practicality

While special relativity *seems* crazy[1], it is validated and tested every day in a variety of modern technologies. Two particularly useful applications are in medical accelerators[2], and the Global Positioning System (GPS). In this exercise, we'll utilize our know how with special relativity to consider both of these cases.

In your answers to the questions, do not simply write down the numerical results. in each case, clearly identify: (a) Which special relativity relationship you are using (b) Exactly what you are measuring (c) Who measures a "proper" quantity (d) Who measures a non-proper quantity.

A▷Medical accelerators

Hospitals and medical facilities with expertise in radiation therapy often have small linear accelerators ("linacs") for use in treating patients. These accelerators typically push protons to high rates of speed. When working properly, they will accelerate *any* charged particle. Suppose you were interested in exploring the use of other charged particles in a hospital linac, in particular particles that decay in a short period of time.

You decide to experiment with the charged kaon, K^+, because it has about half the mass of the proton, so won't do as much damage. The issue with the K^+ is that it decays on short timescales, 1.24×10^{-8} seconds. The linac is 30 meters long in the lab, and the kaons are travelling at a speed of $0.997c$.

▷1◁ You and I are standing next to the linac as the kaons go zipping by. How long do we think it takes the particles to decay?

▷2◁ How far do we think it travels before it decays? Does it make it out of the accelerator and into the patient?

▷3◁ How long do we think it takes for the kaons to travel the full length of the accelerator?

▷4◁ How long does a K^+ think the accelerator is?

▷5◁ From the viewpoint of the kaon, the accelerator is flying past it. How long does the particle think it takes the accelerator to fly by?

B▷Global Positioning System

The largest corrections in GPS come from general relativity, but there is a sizable contribution from special relativity, which we can easily compute the effect of. GPS works on the basis of comparing clocks, and using the time it takes signals to get from the satellites to you to triangulate your position. Because it is based on comparing clock signals, anything that throws off the rate of clocks ticking throws off the accuracy of GPS.

A typical GPS satellite has a speed relative to the surface of the Earth of $v \sim 3900$ m/s.

▷1◁ What is the value of γ?

▷2◁ Your answer above should have been close to 1, as expected since the speed $v \ll c$. Use the binomial approximation for γ to estimate the value of $(1 - \gamma)$. This is the special relativity correction.

▷3◁ Suppose 1 day of time elapses according to the GPS satellite's on board clock. How much time elapses on Earth? That is to say, what is the *difference* between a clock on Earth and the GPS clock?

▷4◁ The difference between the Earth clock and satellite clock is the source of error in GPS positioning. The position error is the distance light can travel in the time difference. What is the positional error after 1 day if special relativity is ignored?

[1] Your instructor's disposition may lead you to believe the same of him!
[2] Smaller cousins to ordinary particle accelerators.