

- ② I discover a 1.0g gold nugget under a pile of homework. It has no net charge. If I remove 1% of its electrons, what is the net charge?
-

First, I determine how many gold atoms are in my nugget, then how many electrons there are. To find out how many atoms there are, I divide the mass of my nugget by the mass of 1 gold atom. Looking at the periodic table a gold atom has a mass of: $M_{Au} = 196.96654 \text{ u/atom}$ where "u" is an "atomic mass unit. So:

$$M_{Au} = \frac{196.96654 \text{ u}}{1 \text{ atom}} \cdot \left(\frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ u}} \right) = 3.27063 \times 10^{-25} \frac{\text{kg}}{\text{atom}}$$

So the number of atoms is:

$$N_{\text{atom}} = \frac{m_{\text{nugget}}}{M_{Au}} = \frac{1.0 \times 10^{-3} \text{ kg}}{3.27063 \times 10^{-25} \text{ kg/atom}} = 3.05752 \times 10^{21} \text{ atoms}$$

Gold has 79 protons and 79 electrons, so the total is:

$$N_{\text{electrons}} = N_{\text{atom}} \cdot 79 = 2.41544 \times 10^{23} \text{ electrons}$$

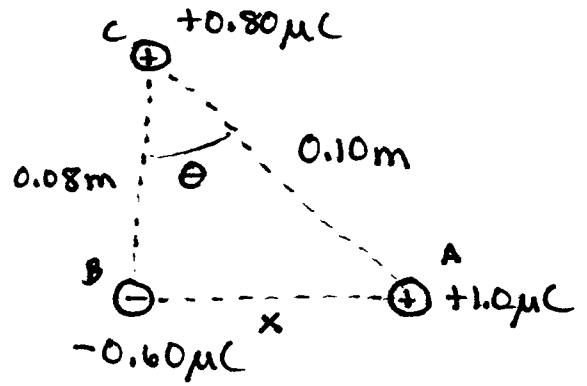
1% of these has a total charge:

$$Q_{\text{TOT}} = N_{\text{electrons}} \cdot e = 2.41544 \times 10^{23} \cdot 1.60 \times 10^{-19} \text{ C} \times 0.01 \\ = 386.5 \text{ C}$$

The net charge left is equal to the net charge stripped so:

$$\boxed{Q_{\text{NET}} = 386.5 \text{ C}} \quad (\text{ANS})$$

- ⑮ In order to join the Intergalactic Federation of Physics, aliens from the Triangulum Galaxy give us the triangular set of charges here, and want to know the NET ELECTRIC FORCE on the $+1.0$ charge.



Like all forces the Coulomb force is a VECTOR. To find the NET FORCE then, it behooves us to work in terms of VECTOR COMPONENTS. To successfully do this I need to complete the geometry of the triangle by computing the distance x (Pythagorean theorem) and the angle θ (definition of COSINE):

$$\begin{aligned} x: (0.10\text{m})^2 &= (0.08\text{m})^2 + x^2 \\ x^2 &= (0.10\text{m})^2 - (0.08\text{m})^2 \\ x &= \sqrt{(0.10\text{m})^2 - (0.08\text{m})^2} = 0.06\text{m} \end{aligned}$$

$$\begin{aligned} \theta: \cos \theta &= \frac{\text{ADJ}}{\text{HYP}} = \frac{0.08\text{m}}{0.10\text{m}} = 0.8 \\ \theta &= \cos^{-1}(0.8) = 36.87^\circ \end{aligned}$$

Next let me compute the magnitude of the individual charges on our $+1.0 \mu\text{C}$ charge:

$$F_{AB} = -k \frac{q_A q_B}{r^2} = \frac{(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(-0.6 \times 10^{-6} \text{C})(1.0 \times 10^{-6} \text{C})}{(0.06\text{m})^2}$$

so/
 $F_{AB} = 1.4983 \text{ N}$ TO LEFT (attractive force) (-x direction)

The remaining force:

$$F_{AC} = \frac{-k q_A q_C}{r^2} \Rightarrow \frac{(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(0.80 \times 10^{-6} \text{ C})(1.0 \times 10^{-6} \text{ C})}{(0.10 \text{ m})^2}$$

$$F_{AC} = 0.7192 \text{ N} \quad \text{DOWN TO RIGHT (repulsive force)}$$

The force F_{AB} ONLY has x components:

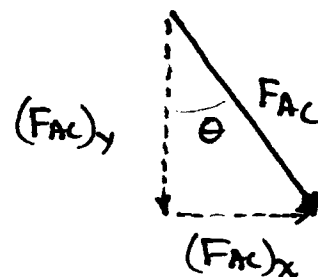
$$(F_{AB})_x = F_{AB} = 1.4983 \text{ N}$$

$$(F_{AB})_y = 0$$

The force F_{AC} has both components:

$$\begin{aligned} (F_{AC})_y &= F_{AC} \cdot \cos \theta \\ &= (0.7192 \text{ N}) \cos 36.87^\circ = 0.5754 \text{ N} \end{aligned}$$

$$\begin{aligned} (F_{AC})_x &= F_{AC} \cdot \sin \theta \\ &= (0.7192 \text{ N}) \sin 36.87^\circ = 0.4315 \text{ N} \end{aligned}$$



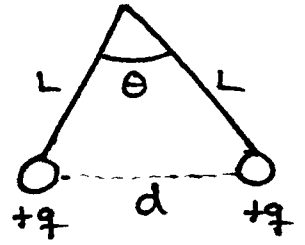
The NET FORCE COMPONENTS are:

$$(F_{NET})_x = (F_{AB})_x + (F_{AC})_x = -1.4983 \text{ N} + 0.4315 \text{ N} = (-)1.0668 \text{ N}$$

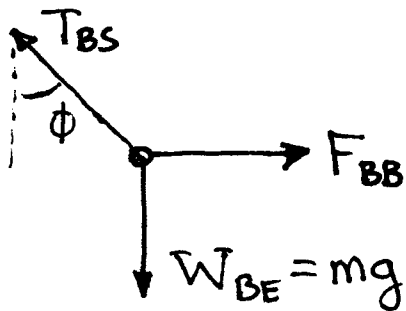
$$(F_{NET})_y = (F_{AB})_y + (F_{AC})_y = 0 - 0.5754 \text{ N} = (-)0.5754 \text{ N}$$

(ANS)

- ⑮ Two styrofoam balls each of mass $m = 9.0 \times 10^{-8} \text{ kg}$ are suspended as shown to give physics students something to think about. Each has a charge $+q$, $L = 0.98 \text{ m}$ and $d = 0.020 \text{ m}$. What is q ?



This can be worked out by looking at the free body diagram for a single ping pong ball. There are 3 forces:



▷ TENSION T_{BS} : unknown

▷ WEIGHT $W_{BE} = mg$

▷ COULOMB FORCE from the other charged ball: $F_{BB} = \frac{kq^2}{d^2}$

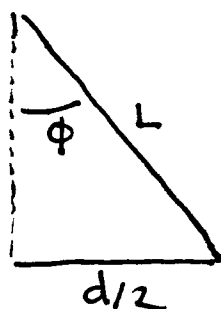
The ball is NOT accelerating, so I know all the forces must sum to zero. This also means the COMPONENTS must sum to zero. So let's start with the VERTICAL COMPONENTS (because I know one of the vertical forces, the weight).

$$(F_{BB})_y = 0 \quad (\text{points purely in } x \text{ direction})$$

$$(T_{BS})_y = T_{BS} \cdot \cos \phi$$

$$(W_{BE})_y = mg$$

In order to complete this calculation I will need to know the angle ϕ .



$$\sin \phi = \frac{\text{OPP}}{\text{HYP}} = \frac{d/2}{L} = \frac{(0.020\text{m})/2}{0.98\text{m}} = 0.0102$$

$$\phi = \sin^{-1}(0.0102) = 0.5847^\circ$$

I know all the components in the Y-direction must sum to zero so:

$$(F_{\text{NET}})_y = (F_{\text{BB}})_y + (T_{\text{BS}})_y - (W_{\text{BE}})_y = 0$$

$$0 = T_{\text{BS}} \cdot \cos \phi - mg \Rightarrow mg = T_{\text{BS}} \cdot \cos \phi$$

$$\text{or/} T_{\text{BS}} = mg / \cos \phi = \frac{(9.0 \times 10^{-8} \text{kg})(9.81 \text{m/s}^2)}{\cos(0.5847^\circ)}$$

$$\boxed{T_{\text{BS}} = 8.83 \times 10^{-7} \text{N}} \quad \text{TENSION IN STRING}$$

Now that I know the tension, I can look at the HORIZONTAL COMPONENTS

$$(W_{\text{BE}})_x = 0 \quad (\text{points purely in y direction})$$

$$(T_{\text{BS}})_x = T_{\text{BS}} \cdot \sin \phi$$

$$(F_{\text{BB}})_x = \frac{kq^2}{d^2} \quad (\text{Coulomb repulsion})$$



I know all the components in the x-direction must sum to zero so:

$$(F_{\text{NET}})_x = (F_{\text{BB}})_x - (T_{\text{BS}})_x + (W_{\text{BE}})_x = 0$$

$$0 = \frac{kq^2}{d^2} - T_{\text{BS}} \cdot \sin \phi + 0$$

$$\text{so// } \frac{kq^2}{d^2} = T_{\text{BS}} \cdot \sin \phi \Rightarrow q^2 = \frac{d^2}{k} (T_{\text{BS}} \cdot \sin \phi)$$

$$\text{or// } q = \left[\frac{d^2}{k} T_{\text{BS}} \cdot \sin \phi \right]^{1/2} = \left[\frac{(0.02\text{m})^2 \cdot 8.83 \times 10^{-7} \text{N} \cdot \sin 0.5847^\circ}{8.99 \times 10^9 \text{Nm}^2/\text{C}^2} \right]^{1/2}$$

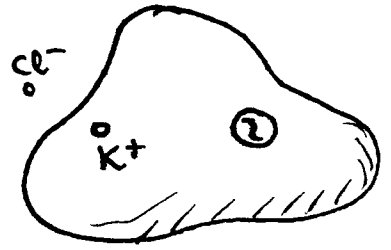
$$\boxed{q = 2.0 \times 10^{-11} \text{C}} \quad (\text{ANS})$$

EXTRA THOUGHT:

If the charge on one elementary charge (proton or electron) is $e = 1.60 \times 10^{-19} \text{C}$ then

$$n_{\text{CHARGES}} = \frac{q}{e} = \frac{2.0 \times 10^{-11} \text{C}}{1.60 \times 10^{-19} \text{C}} = 1.25 \times 10^8$$

- ② An alien cell membrane is 9.0 nm thick. A chlorine Cl^- and potassium K^+ are on opposite sides. What is the electric force on Cl^- due to K^+ ?



The force is a COULOMB ATTRACTION since the two are of opposite charge. So:

$$F_{\text{ClK}} = \frac{k q_{\text{Cl}} q_{\text{K}}}{r^2}$$

The atoms have gained or lost an electron, hence the $+$ or $-$ charges. Thus they have a net charge each of:

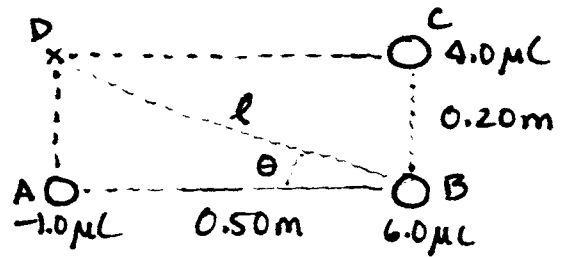
$$q = e = 1.60 \times 10^{-19} \text{ C}$$

so//

$$F = \frac{ke^2}{r^2} = \frac{(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}) (1.60 \times 10^{-19} \text{ C})^2}{(9.0 \times 10^{-9} \text{ m})^2}$$

$$\boxed{F_{\text{ClK}} = 2.84 \times 10^{-12} \text{ N}} \quad (\text{ANS})$$

- 26) The Intergalactic Federation of Physics can't believe we answered the Triangulum Galaxy aliens question (#15) so easily, so pose us with another array of charges from the clever but nerdy aliens of Vega (they're "square"). Given the charges above: (a) What is the E field at the fourth corner? (b) If an $+8.0\mu\text{C}$ charge is placed on the fourth corner, what force does it feel?



As with previous problems, this is a VECTOR PROBLEM. To find the NET E FIELD at the corner, I'll need to work out COMPONENTS. To that end, I'll need l and θ , as indicated in the diagram. I can get l from the PYTHAGOREAN THEOREM and the angle from TANGENT:

$$l^2 = (0.50\text{m})^2 + (0.20\text{m})^2 \Rightarrow l = \sqrt{(0.50\text{m})^2 + (0.20\text{m})^2}$$

$$l = 0.5385\text{m}$$

$$\text{TAN } \theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{0.20\text{m}}{0.50\text{m}} \Rightarrow \theta = \text{TAN}^{-1}(0.20\text{m}/0.50\text{m})$$

$$\theta = 21.8^\circ$$

Now if I label my charges/corners A B C D I can now work out the individual contributions to the net electric field, keeping in mind the magnitude is given by:

$$E = \frac{kq}{r^2}$$



So at point D:

CHARGE AT CORNER A

$$\begin{cases} (E_{AD})_x = 0 \\ (E_{AD})_y = \frac{kq}{r^2} = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(1.0 \times 10^{-6} C)}{(0.20m)^2} = (-) 2.25 \times 10^5 \frac{N}{C} \end{cases}$$

PTS IN $\ominus y$ direction

CHARGE AT CORNER B

$$\begin{cases} (E_{BD})_x = \frac{kq}{r^2} \cdot \cos \theta \\ = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(6.0 \times 10^{-6} C)}{(0.5385m)^2} \cdot \cos 21.8^\circ = (-) 1.73 \times 10^5 \frac{N}{C} \end{cases}$$

PTS IN $(-)x$ direction

$$\begin{cases} (E_{BD})_y = \frac{kq}{r^2} \cdot \sin \theta \\ = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(6.0 \times 10^{-6} C)}{(0.5385m)^2} \cdot \sin 21.8^\circ = 6.91 \times 10^4 \frac{N}{C} \end{cases}$$

CHARGE AT CORNER C

$$\begin{cases} (E_{CD})_x = \frac{kq}{r^2} = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(4.0 \times 10^{-6} C)}{(0.50m)^2} = (-) 1.44 \times 10^5 \frac{N}{C} \\ (E_{CD})_y = 0 \end{cases}$$

PTS IN $(-)x$ DIRECTION

Now that I have all the components, the components of the NET E field are the sum of these.

$$\begin{aligned} (E_{NET})_x &= (E_{AD})_x + (E_{BD})_x + (E_{CD})_x = (-) 3.17 \times 10^5 \frac{N}{C} \\ (E_{NET})_y &= (E_{AD})_y + (E_{BD})_y + (E_{CD})_y = (-) 1.56 \times 10^5 \frac{N}{C} \end{aligned}$$

(ANS) \rightarrow

26 cont....

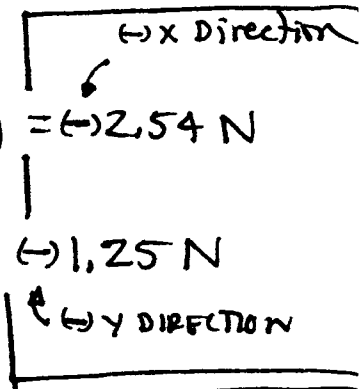
(b) Now that I know the E field, finding the force on any charge q at corner D is a matter of applying the relationship between E and F:

$$E = F/q \Rightarrow F = q \cdot E$$

The components of the force then are:

$$F_x = q \cdot E_x = (8.0 \times 10^{-6} \text{ C})(\leftarrow) 3.17 \times 10^5 \frac{\text{N}}{\text{C}} = \leftarrow 2.54 \text{ N}$$

$$F_y = q \cdot E_y = (8.0 \times 10^{-6} \text{ C})(\leftarrow) 1.56 \times 10^5 \frac{\text{N}}{\text{C}} = \leftarrow 1.25 \text{ N}$$



(Ans)

- ② The electric field across a cellular membrane is $1.0 \times 10^7 \text{ N/C}$ directed INTO the cell. (a) If a pore opens which way do Na^+ ions flow? (b) What is the magnitude of the force on the Na^+ ion?

The force on a + charge is in the same direction as the E field, so the Na^+ will feel a force INTO the cell.

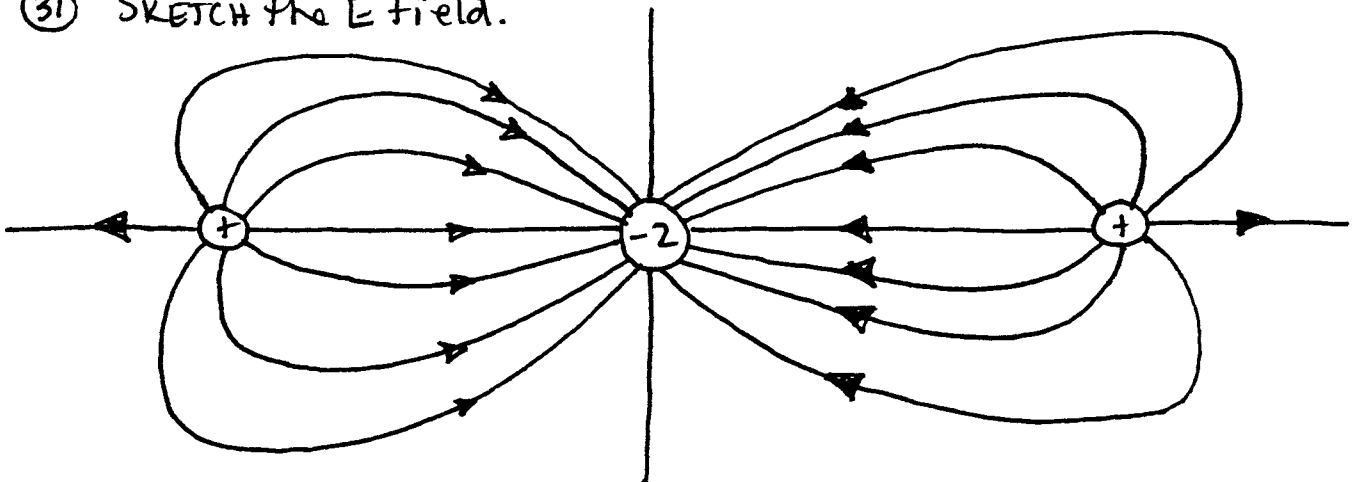
I can get the magnitude of the force from $F = q \cdot E$ remembering that Na^+ is charged with $q = e = 1.6 \times 10^{-19} \text{ C}$

so //

$$F = eE = (1.60 \times 10^{-19} \text{ C})(1.0 \times 10^7 \text{ N/C}) = 1.60 \times 10^{-12} \text{ N}$$

(ANS)

- ③ SKETCH the E field.



- ④7 Protons can be accelerated to high speed to blast cancer cells into oblivion. I have a 4.0m long accelerator that accelerates protons from rest to $1.0 \times 10^7 \text{ m/s}$. What E field is needed to accelerate these protons?

Electric field is related to force by: $E = F/q$

Force is given by Newton II $F = ma$ so: $E = ma/q$

I look up the mass and charge of the proton, so all I need to do is figure out a which I can do from kinematics.

x	4.0m
x_0	0
v	$1 \times 10^7 \text{ m/s}$
v_0	0
a	?
t	?

From my knowledge table, I think a good kinematic eqn is:

$$v^2 = v_0^2 + 2a\Delta x$$

which can easily be solved for a

in terms of things I know:

$$2a\Delta x = v^2 - v_0^2 \Rightarrow a = \frac{1}{2\Delta x} (v^2 - v_0^2)$$

$$\text{or} \quad a = \frac{1}{2 \cdot (4\text{m})} \left((1 \times 10^7 \text{ m/s})^2 - (0 \text{ m/s})^2 \right) = 1.25 \times 10^{13} \text{ m/s}^2$$

So the electric field is:

$$E = \frac{ma}{q} = \frac{(1.25 \times 10^{13} \frac{\text{m}}{\text{s}^2}) (1.673 \times 10^{-27} \text{ kg})}{1.60 \times 10^{-19} \text{ C}} = \boxed{1.31 \times 10^5 \frac{\text{N}}{\text{C}} = E} \quad (\text{Ans})$$

- 57) A $0.890 \mu\text{C}$ charge is placed at the center of the CUBE OF INFINITE CONDUCTION. What is the flux thru one side of the cube?
-

Since the charge is completely enclosed, Gauss' Law gives us the TOTAL flux thru the cube:

$$\Phi_{\text{TOT}} = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{0.890 \times 10^{-6} \text{C}}{8.854 \times 10^{-12} \text{C}^2/\text{Nm}^2} = 1.0 \times 10^5 \frac{\text{Nm}^2}{\text{C}}$$

Since all six sides of the cube are identical, the flux thru each side is the same:

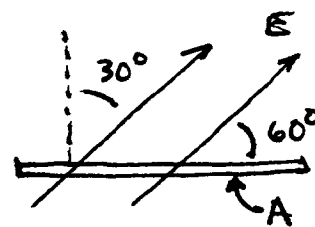
$$\Phi_{\text{TOT}} = 6 * \Phi_{\text{SIDE}} \Rightarrow \Phi_{\text{SIDE}} = \frac{1}{6} \Phi_{\text{TOT}}$$

or,

$\Phi_{\text{SIDE}} = 1.68 \times 10^4 \frac{\text{Nm}^2}{\text{C}}$

 (ANS)

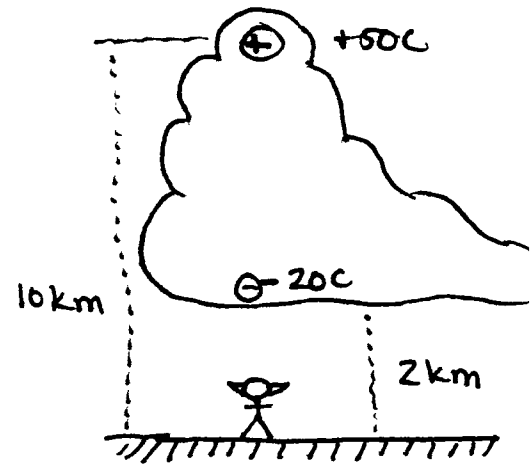
- 59) Uniform Electric field lines cross an area A making a 60° angle with the plane. What is flux?



Flux is normally described in terms of the angle the field makes with the perpendicular, 30° in this case. So:

$$\Phi = EA \cos \theta = EA \cos 30^\circ = \frac{\sqrt{3}}{2} EA = \Phi \quad (\text{ANS})$$

- 84) As a class project, you decide to let your professor stand in a thunderstorm to measure the E field. The situation is shown at right. (a) What is the magnitude and direction of E at your professor? (b) What sign of charge accumulates at your professor's feet if Earth is a conductor?



- (a) The E from the +50C points DOWN and the E from the -20C points UP. The direction of the NET field depends on the relative magnitudes.

$$E_+ = \frac{kq}{r^2} = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(50C)}{(10000m)^2} = 4.495 \times 10^3 \frac{N}{C} \quad \text{DOWN}$$

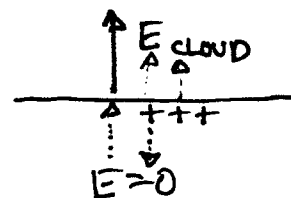
$$E_- = \frac{kq}{r^2} = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(20C)}{(2000m)^2} = 4.495 \times 10^4 \frac{N}{C} \quad \text{UP}$$

So the Net field is:

$$E_{NET} = E_{UP} - E_{DOWN} = 4.05 \times 10^4 \frac{N}{C} \quad \text{UP}$$

(ANS)

- (b) Under the ground $E=0$ to be a conductor, so the ground charges must be



POSITIVE to cancel the cloud field in the ground. This increases the E field above the ground.