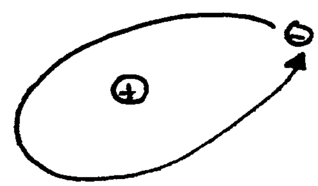
Φ2020
CH 17: 2, 10, 17, 21, 25, 29, 32, 33, 39, 44, 50, 61, 89



② A hydrogen atom consists of a proton with an orbiting electron 0.0529 nm away. (a) What is the electric potential energy? (b) What does the sign mean?

Electric potential energy is given by: $U_E = \frac{k q_1 q_2}{r}$

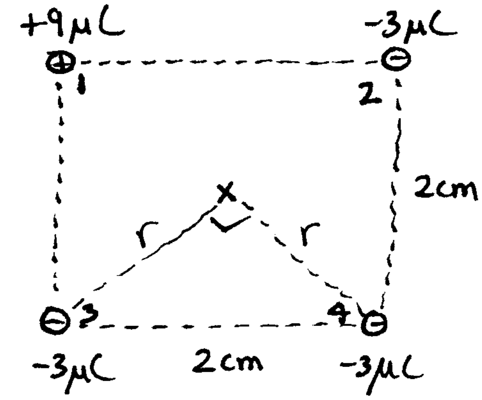
so//

$$U_E = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(1.6 \times 10^{-19} C)(-1.6 \times 10^{-19} C)}{(0.0529 \times 10^{-9} m)}$$

$$\boxed{U_E = (-) 4.35 \times 10^{-18} J}$$ (ANS)

(b) The sign (NEGATIVE) is an indicator that the proton and electron are BOUND (an attractive force keeps them together). To break them apart (make $U_E = 0$) I would have to DO WORK ON THE SYSTEM (I.E. ADD ENERGY to the system).

(10) The aliens from Quadriplex Prime have once again seized Earth and in exchange for its release demand you consider the square of charges shown and tell them the electric field and potential at the center.



Calculations of both the potential and the E field require knowing the distance from the charges to the center of the square. The distance is the same in each case since this is a square. Imagine a right triangle (shown) with two sides of length r and one of length 0.02m then:

$$r^2 + r^2 = (0.02m)^2 \quad \text{PYTHAGOREAN THM}$$
$$2r^2 = 4.0\times10^{-4}\,m^2$$
$$r^2 = 2.0\times10^{-4}\,m^2$$
$$\boxed{r = 1.41\times10^{-2}\,m}$$

First, we compute the potential. The total potential is simply the sum of all the potentials; the potential due to an individual charge q a distance r away is

$$V = \frac{kq}{r} \Rightarrow V_{TOT} = V_1 + V_2 + V_3 + V_4$$

but charges $q_2 = q_3 = q_4 = (-)3\mu C$ and all are $r = 0.0141m$ away so $V_2 = V_3 = V_4$ so:

$$V_{TOT} = V_1 + 3V_2$$

→

sol/ $V_{TOT} = \frac{kq_1}{r} + 3\,\frac{kq_2}{r}$

$= \frac{k}{r}\left(q_1 + 3q_2\right)$

$= \frac{k}{r}\left(9.0\times10^{-6}C - 3\cdot 3.0\times10^{-6}C\right) = \boxed{0\ V}$

(ANS)

Next I consider the electric field, a VECTOR, so I must consider both magnitude and direction. Let's consider pairs of charges:

CHARGE 2 & CHARGE 3 : $\vec{E}_2$ points toward q_2 ; $\vec{E}_3$ points toward q_3, in the EXACT OPPOSITE DIRECTION of $\vec{E}_2$. Since $q_2 = q_3$, and they are both the same distance from the center of the square, the magnitudes $E_2 = E_3$. Equal magnitude vectors pointing in OPPOSITE directions sum to ZERO.

CHARGE 1 & CHARGE 4 : $\vec{E}_1$ points AWAY from charge 1 and $\vec{E}_4$ points TOWARD charge 4. So both $\vec{E}_1$ and $\vec{E}_4$ point toward q_4, so the E fields add together. The magnitudes are:

$$E_1 = \frac{kq_1}{r_1^2} = \frac{(8.99\times10^{9}\,\frac{Nm^2}{C^2})(9.0\times10^{-6}C)}{(0.0141m)^2} = 4.07\times10^{8}\,\frac{N}{C}$$

$$E_4 = \frac{kq_4}{r_4^2} = \frac{(8.99\times10^{9}\,\frac{Nm^2}{C^2})(3.0\times10^{-6}C)}{(0.0141m)^2} = 1.36\times10^{8}\,\frac{N}{C}$$

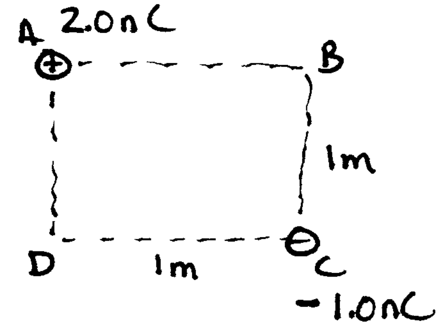
→

Note I've not used $-3.0\mu C$, but $+3.0\mu C$. Since I'm computing the MAGNITUDE I've taken an absolute valve. the (-) sign only tells me about the direction of $\vec{E}$, which I've already reasoned out.

so//

$$E_{TOT} = E_1 + E_4 = \boxed{5.43 \times 10^8 \frac{N}{C} \text{ TOWARD CHARGE 4}}$$ (ANS)

I could have also worked out the x and y components for E.



(17) After successfully winning back the Earth in problem (10), a rebel faction of Quadriplexi, the DIAGOLONS demand you proove yourself a second time, in case the first time was a fluke. What is the potential at corner B?

The potential due to a single charge is: $V = \frac{kq}{r}$

For q_A and q_C, they are equidistant from B, so $r_A = r_C = 1.0\text{m}$. So the total potential at B is

$$V_{TOT} = V_A + V_C = \frac{kq_A}{r_A} + \frac{kq_C}{r_C}$$

$$= \frac{k}{r}(q_A + q_C)$$

$$= \frac{\left(8.99\times10^{9}\ \frac{Nm^2}{C^2}\right)}{1.0\text{m}} \cdot \left(2.0\times10^{-9}C - 1.0\times10^{-9}C\right)$$

$$= 8.99\ \frac{Nm}{C} = \boxed{8.99\ V}$$ (ANS)

(21) Show $1\,N/C = 1\,V/m$.

We defined: $1V = 1\dfrac{J}{C} = 1\dfrac{Nm}{C} = 1\dfrac{kg\frac{m}{s^2}\cdot m}{C}$

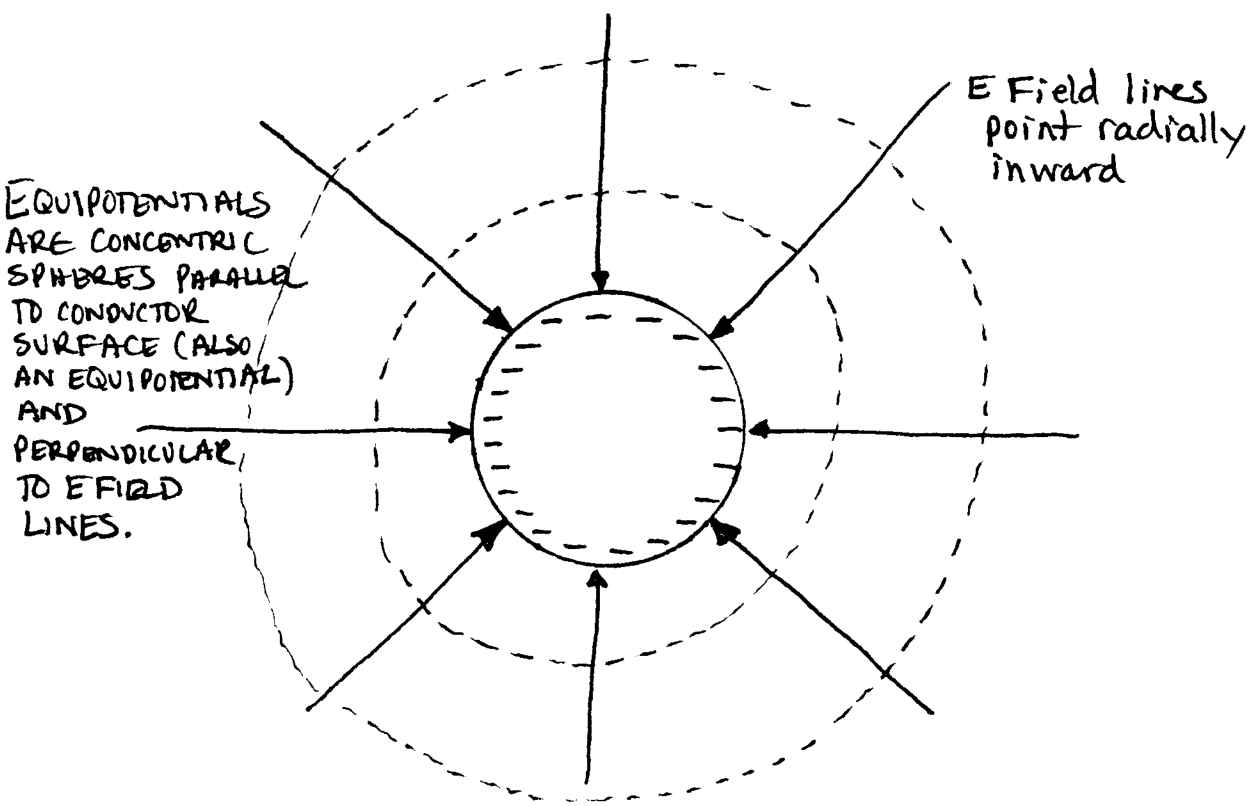
$= 1\dfrac{kg\,m^2}{C\cdot s^2} \Rightarrow 1\dfrac{V}{m} = 1\dfrac{kg\,m}{C\cdot s^2}$

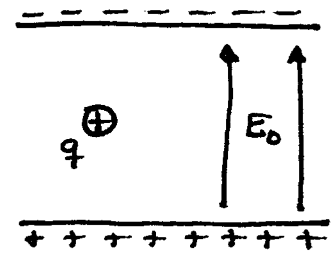
Similarly: $1\dfrac{N}{C} = 1\dfrac{kg\,m/s^2}{C} = 1\dfrac{kg\cdot m}{C\cdot s^2}$

SAME.

Q.E.D.

(25) Draw the electric field and equipotentials around a hollow conducting negatively charged sphere.





(29) A positive charged oil drop enters the electric field $E_0 = 3000\ V/m$ between two plates. The electric force is 9.6×10^{-16} N. What is the charge on the drop in terms of e?

The base proton/electron charge is $e = 1.60 \times 10^{-19}$ C.

Remember that electric field and force are easily related by the charge q experiencing the force:

$$E = \frac{F}{q} \Rightarrow q = \frac{F}{E}$$

so//

$$q = \frac{9.6 \times 10^{-16}\ N}{3000\ V/m} = \frac{9.6 \times 10^{-16}\ N}{3000\ N/C} = 3.2 \times 10^{-19}\ C$$

or//

$$\frac{q}{e} = \frac{3.2 \times 10^{-19}\ C}{1.6 \times 10^{-19}\ C} = 2 \Rightarrow \boxed{q = 2e}$$ (ANS)

(32) An electron accelerates from rest thru a potential difference ΔV. If final speed is 7.26×10^6 m/s what is ΔV? Include the correct sign.

The electron is accelerating from REST. Initially it has ONLY potential energy U_E, and it is all converted into kinetic energy K_E so: $U_{Ei} = K_{Ef}$.

I can relate U_E to potential by: $U_E = qV$

so//

$$U_{Ei} = K_{Ef} \Rightarrow qV = \frac{1}{2}mv^2$$

so//

$$V = \frac{mv^2}{2q} = \frac{(9.109 \times 10^{-31}\,kg)(7.26 \times 10^6\,m/s)^2}{2 \cdot (1.662 \times 10^{-19}\,C)}$$

$$= 149.8\,V$$

The electron is ACCELERATING, so it moves from LOW POTENTIAL to HIGH POTENTIAL, thus ΔV is positive.

$$\boxed{\Delta V = 150\,V}$$ (ANS)

(33) An electron's speed decreases from 8.50×10^6 m/s to 2.50×10^6 m/s due to an electrical force. (a) Is the electron moving from high to low potential or vice versa? (b) Across what potential difference did it travel?

(a) The electron is DECELERATING, so it is gaining potential energy, therefore (for electrons) it is moving toward LOWER POTENTIAL (eg it is approaching a negative plate).

(b) Here the change in K_E is equivalent to the change in U_E : $\Delta K_E = \Delta U_E$. If I assume $U_{Ei} = 0$ then $U_{Ef} = qV$ and I may write:

$$|\Delta K_E| = \left|\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right| = |qV|$$

or//

$$V = \frac{1}{2}m\,\frac{|v_f^2 - v_i^2|}{|q|}$$

$$= \frac{(9.109 \times 10^{-31}\,\text{kg})\left((8.50 \times 10^6\,\text{m/s})^2 - (2.50 \times 10^6\,\text{m/s})^2\right)}{2 \cdot (1.602 \times 10^{-19}\,\text{C})}$$

$$= 186.9\ \text{V}$$

Since the electron is moving from HIGH to LOW potential ΔV is negative:

$$\boxed{\Delta V = (-)\,187\ \text{V}}$$ (ANS)

(39) I have a large 10.2 μF capacitor, and I want to lower the potential difference across its plates by 60.0 V. How much Charge must I remove?

Charge is related to capacitance and potential difference by:

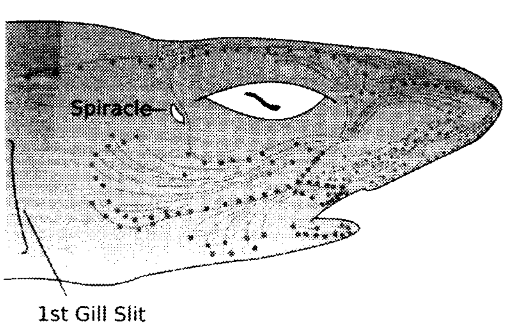
$$Q = C\Delta V = (10.2\times10^{-6}\,\mathrm{F})(60.0\,\mathrm{V}) = \boxed{6.12\times10^{-4}\,\mathrm{C}}$$ (ANS)

(44) A shark's electroreceptors ("AMPULLAE OF LORENZINI") can detect electric fields as small as 1.0 μV/m. Suppose you have a 1.5 V battery and a parallel plate capacitor, how for apart must the plates be to gererate this size E field?

The battery makes the plates of the capacitor $\Delta V = 1.5\,\mathrm{V}$. I relate that potential difference to E by the plate separation d: $\Delta V = E\cdot d$

so//

$$d = \frac{\Delta V}{E} = \frac{1.5\,\mathrm{V}}{1.0\times10^{-6}\,\frac{\mathrm{V}}{\mathrm{m}}} = 1.5\times10^{6}\,\mathrm{m} = \boxed{1500\,\mathrm{km}}$$ (ANS)



LOCATION OF ELECTRORECEPTORS IN A SHARK'S HEAD.

(50) The required electric field to generate a lightning strike in damp air is 3.33×10^5 V/m. If a thundercloud has a potential 1.00×10^8 V less than the surface of the Earth, what is the max height above the ground the cloud can be to have a lightning strike.

I relate electric field E to potential difference ΔV by: $\Delta V = Ed$. Here ΔV is the potential difference between the cloud and Earth, 1.0×10^8 V.

so//

$$\Delta V = Ed \Rightarrow d = \frac{\Delta V}{E} = \frac{1.0 \times 10^8 \text{ V}}{3.33 \times 10^5 \frac{\text{V}}{\text{m}}} \doteq \boxed{300.3 \text{ m}}$$ (ANS)

(61) I model the base of a thundercloud as the negative plate in a capacitor and the Earth as the positive plate. The cloud measures 4.5 km x 2.5 km and is 550 m above the Earth, carrying 18C of charge. (a) What is the capacitance? b) What is the stored energy?

(a) Capacitance may be computed from physicad dimensions:

$$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.854\times10^{-12}\,\frac{C^2}{Nm^2}\right)(4500\,m \cdot 2500\,m)}{550\,m} = \boxed{1.81\times10^{-7}\,F}$$

(ANS)

(b) The energy stored in a capacitor is given by:

$$U = \frac{Q^2}{2C} = \frac{(18C)^2}{2(1.81\times10^{-7}F)} = \boxed{8.95\times10^{8}\,J}$$

(ANS)

(89) An axon (nerve fiber) has part of its membrane positively charged on exterior and negatively charged on the interior. The thickness is 4.4 nm and the dielectric constant is $K=5$. If the axon is modeled as a capacitor with area $5\,\mu m^2$, what is the capacitance?

Capacitance can be computed from physical dimensions as:

$$C = K\epsilon_0 \frac{A}{d} = \frac{5\cdot\left(8.854\times10^{-12}\,\frac{C^2}{Nm^2}\right)\left(5\,\mu m^2\right)\left(\frac{1\,m}{10^6\,\mu m}\right)^2}{4.4\times10^{-9}\,m}$$

$$\boxed{C = 5.03\times10^{-14}\,F}$$ (ANS)