Φ2020: CH 22: 9, 13, 15, 34, 39, 40
CH 23: 8, 16, 17, 23, 39, 40, 42, 48

(9) ROCK N ROLL SCI FI FM 90.9 MHz broadcasts at what λ?

Wavelength and frequency are related by the speed of light:

$$c = \lambda f \rightarrow \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \, m/s}{(90.9 \times 10^6 \, Hz)} = \boxed{3.30 \, m} \text{ (ANS)}$$

(13) A frequency of 2:1 is called a frequency octave. (a) Approx how many octaves are covered by visible light? (b) Approx how many octaves are covered by microwaves?

From Fig 22.7, visible light spans $f_{Lo} \simeq 4.3 \times 10^{14}$ Hz to $f_{HI} \simeq 7.5 \times 10^{14}$ Hz. The ratio is:

$$f_{HI}/f_{LO} = (7.5 \times 10^{14} \, Hz)/(4.3 \times 10^{14} \, Hz) = 1.7 \simeq \boxed{1 \text{ OCTAVE}} \text{ (ANS)}$$

Microwaves span from $f_{LO} \simeq 10^9$ Hz to $f_{HI} \simeq 10^{12}$ Hz

so,

$$f_{HI}/f_{LO} = (10^{12} \, Hz)/(10^9 \, Hz) = 1000 \simeq 2^{10} \simeq \boxed{10 \text{ OCTAVES}} \text{ (ANS)}$$

(15) Current oscillates at 60 Hz. In 1 cycle, how far does the EM radiation generated by the current travel? This distance is λ for a 60 Hz wave. How does it compare to LA-NYC distance?

The time T for one cycle is: $T = \frac{1}{f} = \frac{1}{60\,Hz} = 0.0167\,s$.

The distance traveled is found from the speed of light:

$$C = \Delta x / T \rightarrow \Delta x = C \cdot T = (3.0 \times 10^8\, m/s)(0.0167\,s) = \boxed{5.0 \times 10^6\, m}$$ (ANS)

This is farther than the $4.2 \times 10^6\,m$ separation of LA-NYC.

(34) A star 14×10^6 lyr from Earth has an intensity at Earth of $4 \times 10^{-21}\, W/m^2$. What rate does the star radiate energy?

The rate of energy radiated is Power, related to intensity by:

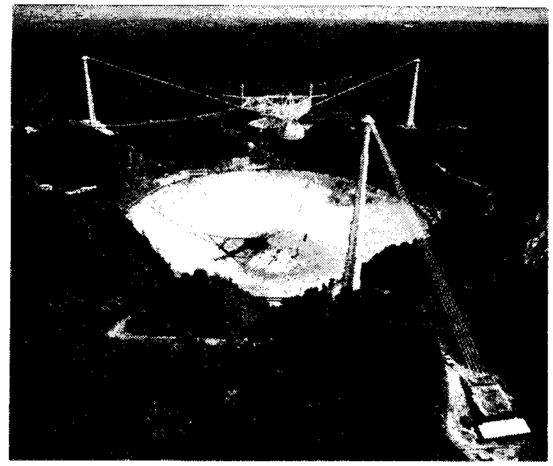
$$P = I \cdot A$$

The area of interest is the surface area of a sphere 14 million lyr in radius, so:

$$A = 4\pi r^2 = 4\pi \cdot \left(14 \times 10^6\, lyr \cdot 9.5 \times 10^{15} \frac{m}{lyr}\right)^2 = 2.22 \times 10^{47}\, m^2$$

so/

$$P = IA = \left(4 \times 10^{-21} \frac{W}{m^2}\right)\left(2.22 \times 10^{47}\, m^2\right) = \boxed{8.90 \times 10^{26}\, W}$$ (ANS)



(39) the Arecibo Telescope is 305m in diameter. The faintest signal it can detect is 10^{-26} W/m². (a) What is the average power received for these minimally detectable waves? (b) What is average power at the Earth's surface? (c) What are Rms E & B fields?

(a) This depends on the area of Arecibo: $A = \pi r^2 = \pi (305m)^2 = 2.9 \times 10^5 m^2$

so//

$$P = IA = (10^{-26} W/m^2)(2.9 \times 10^5 m^2) = \boxed{2.9 \times 10^{-21} W}$$ (ANS)

(b) This depends on the cross sectional area of Earth:

$$A = \pi r_E^2 = \pi (6.37 \times 10^6 m)^2 = 1.27 \times 10^{14} m^2$$

so//

$$P = IA = (10^{-26} W/m^2)(1.27 \times 10^{14} m^2) = \boxed{1.3 \times 10^{-12} W}$$ (ANS)

(c) The field strength can be related to intensity thru energy density:

$$u = \epsilon_0 E_{rms}^2 = I/c \rightarrow E_{rms}^2 = I/c \cdot \epsilon_0 \rightarrow E_{rms} = \sqrt{I/c \cdot \epsilon_0}$$

so//

$$E_{rms} = \left[(10^{-26} W/m^2)/(3.0 \times 10^8 m/s)(8.85 \times 10^{-12} C^2/Nm^2)\right]^{1/2} = \boxed{1.94 \times 10^{-12} \frac{V}{m}}$$

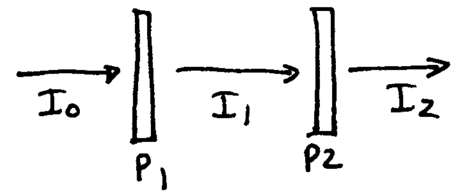
(ANS)

Erms and Brms are simply related:

$$E_{rms} = c \cdot B_{rms} \rightarrow B_{rms} = \frac{E_{rms}}{c} = \frac{(1.94 \times 10^{-12} V/m)}{(3.0 \times 10^8 m/s)} = \boxed{6.47 \times 10^{-21} T}$$

(ANS)

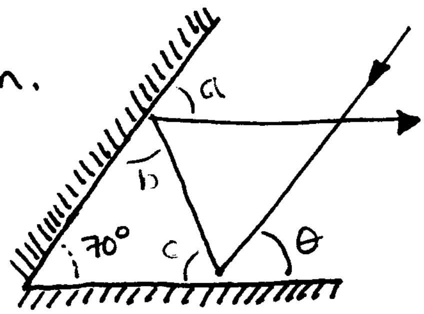
(40) Unpolarized light passes thru 2 polarizers rotated 45° with respect to each other. What fraction of light is transmitted?



The situation is shown to right. If I_0 is unpolarized, then I_1 IS Polarized and $I_1 = \frac{1}{2} I_0$. We get I_2 from I_1 using the Law of Malus:

$$I_2 = I_1 \cdot \cos^2\theta = \frac{1}{2} I_0 \cdot \cos^2 45° = \frac{1}{2} I_0 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 = \boxed{\frac{1}{4} I_0}$$ (ANS)

(8) Two plane mirrors intersect at 70° as shown. For what angle θ is the outgoing ray parallel to the first mirror?

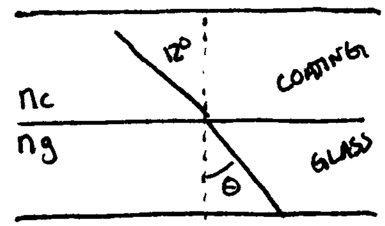


Consider the angles (a) and (b) on the diagram. Since the outgoing ray is parallel to the base mirror, $a = 70°$.

The law of reflection then says $a = b$, so $b = 70°$. The law of reflection also says $c = \theta$. Since the sum of angles in a triangle is 180° I can write:

$$180° = 70° + b + c = 70° + 70° + \theta$$

or, $$\theta = 180° - 140° = \boxed{40°}$$ (ANS)



(16) A glass lens has a scratch resistant coating. In the coating $v = 0.80c$ and in glass $v = 0.67c$. Going from the coating to the glass a ray makes a 12° angle. What is the transmitted angle θ?

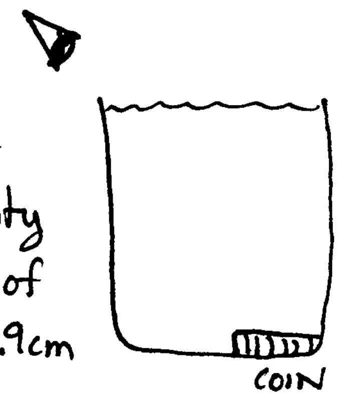
The index of refraction is related to speed by : $n = \frac{c}{V}$
I can use this to rewrite Snell's law as :

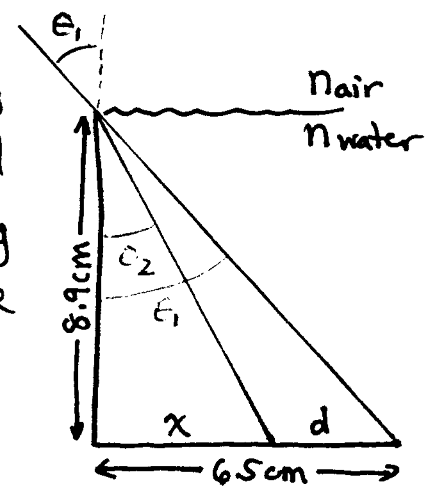
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \longrightarrow \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2}$$

Sol/

$$\theta_2 = \sin^{-1}\left[\frac{V_2}{V_1} \cdot \sin\theta_1\right] = \sin^{-1}\left[\frac{0.67}{0.80} \cdot \sin 12°\right] \doteq \boxed{10°} \text{ (ANS)}$$

(17) A coin of diameter d lies on the bottom of a mug as shown. When the mug is empty I can't see the coin, and when it is full of water I can. Mug is 6.5cm across and 8.9cm tall. How big is the coin?





This problem is based on 2 triangles shown at right, which can be used with SNELL's LAW. θ_1 is the viewing angle, given by the geometry of the empty mug:

$$\tan\theta_1 = \left(\frac{6.5\text{cm}}{8.9\text{cm}}\right)$$

or// $\theta_1 = \tan^{-1}(6.5/8.9) = 36.14°$

SNELL's Law tells me: $n_{air} \cdot \sin\theta_1 = n_{water} \cdot \sin\theta_2$

or// $\theta_2 = \sin^{-1}\left[\frac{n_{air}}{n_{water}} \sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.33} \sin 36.14°\right] = \sin^{-1}[0.4425]$

$= 26.26°$

So in my sketch: $x = 8.9\text{cm} \cdot \tan\theta_2 = 8.9\text{cm} \cdot \tan(26.26°) = 4.39\text{cm}$

so// $d = 6.5\text{cm} - x = 6.5\text{cm} - 4.4\text{cm} = \boxed{2.1\text{cm} = d}$ (ANS)

(23) Calculate the critical angle for total internal reflection for a diamond in AIR and WATER. Why do diamonds sparkle less underwater?

Total internal reflection occurs for incident angles θ_i greater than the critical angle given by:

$$\theta_c = \sin^{-1}\left[\frac{n_T}{N_i}\right]$$

Diamonds sparkle when light is total internal reflected, and returns to the viewer rather than passing thru the diamond and out the far side. For light leaving diamond:

DIAMOND · AIR: $n_T = n_{air} = 1.000$ $n_i = n_{DIAMOND} = 2.419$

so//

$$\theta_c = \sin^{-1}\left[\frac{1.000}{2.419}\right] = 24.42°$$

DIAMOND · WATER: $n_T = n_{WATER} = 1.333$ $n_i = n_{DIAMOND} = 2.419$

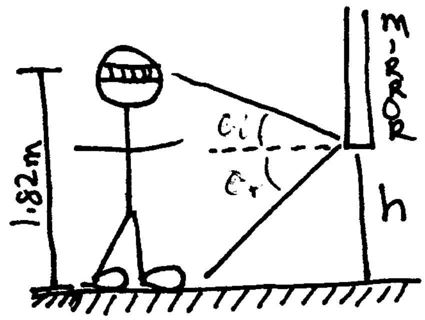
so//

$$\theta_c = \sin^{-1}\left[\frac{1.333}{2.419}\right] = 33.44°$$

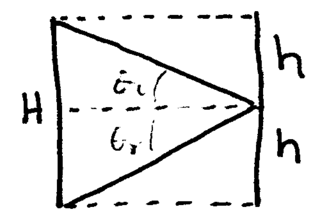
In the second case, the critical angle is LARGER. Sparkle occurs when light from the far surfaces reflect back to the viewer, and a larger critical angle means fewer rays can do so.

(39) DANNY "I WANNA BE LIKE MIKE" JONES' eyes are 1.82 m from the floor. He is 1.96m tall, and his mirror is 0.98m long. How high above the floor must the edge of the mirror be for Danny to see his feet?

From the diagram at right and THE LAW OF REFLECTION, $\theta_i = \theta_r$. The only way for this to be true is if $H = 2h$ or//



$$h = \tfrac{1}{2}H = \tfrac{1}{2}(1.82m) = \boxed{0.91m}$$ (ANS)



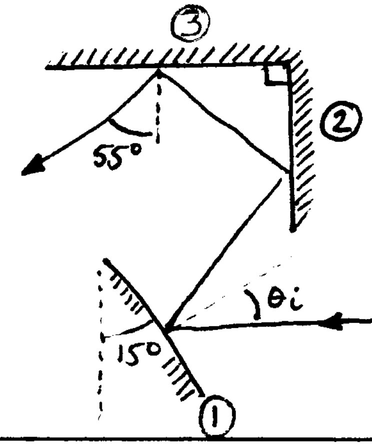
(40) Suppose I build a Lego rose and place it 0.250m in front of a plane mirror. If I stand 2.0m from the mirror, how far away is the image of the rose?

The image is as far behind the mirror as the object is in front, in this case: $d_i = 0.25m$.

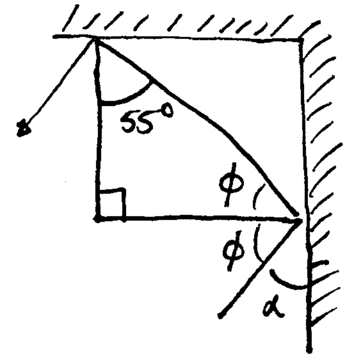
If I am 2m from the mirror then the rose image is:

$$2.0m + 0.25m = \boxed{2.25m \text{ away}}$$ (ANS)

(42) If you and I are trapped in a mirror maze at PHYSKS MOUNTAIN AMUSEMENT PARK, and can see each other in the mirror geometry shown at right, what is the angle θ_i?



This is simply a process of working thru triangles. Since the 55° off mirror (3) is a reflection, consider the ray between mirror (3) and (2):



Since sum of angles in a triangle is 180° then:

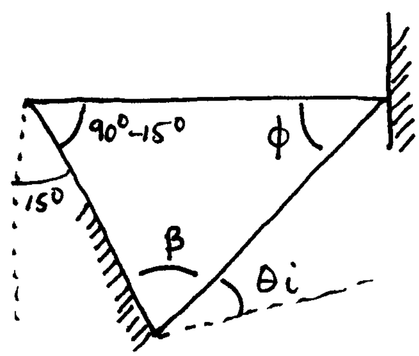
$$180° = 55° + 90° + \phi$$

or// $\phi = 180° - 90° - 55° = 35°$

From the geometry of the reflection on (2): $\phi + \alpha = 90°$

or// $\alpha = 90° - \phi = 90° - 35° = 55°$

Now consider mirror (2) to (1). As shown to the right, I know the upper angle since mirror (1) is tipped 15° from vertical, so:



$$180° = \phi + \beta + (90° - 15°)$$

or// $\beta = 180° - 35° - (90° - 15°) = 70°$

I also know that $\beta + \theta_i = 90°$, so at last I have:

$$\theta_i = 90° - \beta = 90° - 70° = \boxed{20°}$$ (ANS)

(48) I used to be a dental hygenist, but an ankle injury diverted me into physics. I used to have a spiffy convex mirror. When it was 1.20cm from a tooth, I saw an UPRIGHT 3x MAGNIFIED image. What is the focal length and radius of curvature for this mirror?

Magnification is given by: $m = (-)\frac{d_i}{d_o}$

The image distance then is: $d_i = (-) m \cdot d_o$

$= (-) 3 \cdot 1.20\text{cm} = (-) 3.6\text{cm}$

The (-) indicates the image is BEHIND the mirror.

The mirror equation is: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

so//

$$\frac{1}{f} = \frac{1}{1.20\text{cm}} - \frac{1}{3.60\text{cm}} = \frac{0.556}{\text{cm}} \Rightarrow \boxed{f = 1.8\text{cm}}$$ (ANS)

The radius of curvature is $2 \times f$ so: $\Rightarrow \boxed{R = 3.6\text{cm}}$ (ANS)