(5) A STARFLEET issue ROLEX moves at 2.0×10^8 m/s with respect to STARFLEET HQ. At HQ, 12 h elapses, so how much time passes on the Rolex?

First, we must determine which time is proper time, so:

Q: What are we trying to measure?

A: Elapsed time on the Rolex.

The events that define this measurement are the start and end positions of the Rolex. PROPER TIME is measured in the frame where the events occur at the SAME SPATIAL LOCATION (events are at rest). In this case that is the frame of the Rolex — the ROLEX is measuring the proper time in this instance. That means the ROLEX will run slower than clocks at HQ:

$$\Delta\tau = \frac{\Delta t}{\gamma} \qquad \gamma = \left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} = \left[1-\left(\frac{2}{3}\right)^2\right]^{-1/2} = 1.34$$

so//

$$\Delta\tau = \frac{12^h}{1.34} = \boxed{8.9^h = \Delta\tau}$$ (ANS)

⑥ A pion ($\pi^{\pm}$) has a lifetime of 25 ns. A beam of pions is travelling at 0.60c. (a) What do we measure for lifetime in lab? (b) How far do we in the lab think it goes?

The pion measures lifetime according to the internal clock it carries with it — the "ticks" of that internal clock are are rest (occur in the same spatial location) in the pions frame, so the PION MEASURES PROPER LIFETIME. The pion's clock runs slower than the lab clocks:

$$\Delta t = \gamma \Delta \tau \qquad \gamma = \left[1 - \left(\tfrac{v}{c}\right)^2\right]^{-1/2} = \left[1 - (0.6)^2\right]^{-1/2} = 1.25$$

so//

$$\Delta t = (1.25)(25\,\text{ns}) = \boxed{31.25\,\text{ns}}$$

LIFETIME OBSERVED IN LAB FRAME (ANS)

(b) Everyone defines speed as: $V = \Delta S / \Delta T$ where ΔS and ΔT are specified by a particular observer. In the lab:

$$\Delta T = \Delta t = 31.25\,\text{ns} \qquad \text{so//} \quad V = \frac{\Delta S}{\Delta T} \Rightarrow \Delta S = V \cdot \Delta T$$

or//

$$\Delta S = (0.6 \cdot c)(31.25 \times 10^{-9}\,\text{s})$$
$$= 0.6 \cdot (3.0 \times 10^{8}\,\text{m/s}) \cdot (31.25 \times 10^{-9}\,\text{s})$$

$$\boxed{\Delta S = 5.62\,\text{m}}$$ (ANS)

(13) An Imperial Star Destroyer travels toward Earth at 0.97c. The Stormtroopers stand with torsos parallel to the trajectory (IE they are laying down on the job). According to observers on Yavin the troopers are 0.50m tall and 0.5m wide. If Darth Vader is on the ISD with the Stormies, how tall and wide does he think they are?

The ISD is at rest with respect to the Stormies, so ISD observers (Vader) measure PROPER LENGTHS. This means that observers on Yavin see CONTRACTED LENGTHS along the direction of motion.

SO//
$$\Delta L = \frac{\Delta \lambda}{\gamma} \Rightarrow \Delta\lambda = \gamma \Delta L$$

and// $$\gamma = \left[1-\left(\tfrac{v}{c}\right)^2\right]^{-1/2} = \left[1-(0.97)^2\right]^{-1/2} = 4.11$$

SO// $$\Delta\lambda = (4.11)(0.5\,m) = \boxed{\begin{array}{l} 2.06\,m \text{ TALL} \\ 0.5\,m \text{ WIDE} \end{array}}$$ (ANS)

(15) A cosmic ray buzzes the local football game, flying from one goal line to the other at 0.5c. (a) The puny Earthers measure the distance between goal lines to be 91.5m; what does the particle observe? (b) How long do the Earthers think it took the cosmic ray to fly downfield? (c) How long does the particle think it takes?

(a) The Earthers are at rest by the field, so they measure its PROPER LENGTH: $\gamma = [1-(\frac{v}{c})^2]^{-1/2} = [1-(0.5)^2]^{-1/2} = 1.16$

so//

$$\Delta L = \frac{\Delta\lambda}{\gamma} = \frac{91.5m}{1.16} \doteq \boxed{79.2m}$$ (ANS)

(b) If $v = \Delta S/\Delta T$ then we Earthers measure

$$\Delta T = \Delta S/v = \frac{91.5m}{0.5 \cdot 3\times10^8 m/s} \doteq \boxed{6.1\times10^{-7}s = \Delta t}$$ (ANS)

(c) There are two ways to get this answer:

⟨1⟩ The ends of the field fly by the particle; according to the particle these events are at the SAME PLACE (at rest) So it measures PROPER TIME:

$$\Delta t = \gamma\Delta\tau \rightarrow \Delta\tau = \Delta t/\gamma = (6.1\times10^{-7}s)/1.16 \doteq \boxed{5.28\times10^{-7}s}$$ (ANS)

⟨2⟩ The particle measures a 79.2m field, so:

$$v = \frac{\Delta S}{\Delta T} \rightarrow \Delta T = \frac{\Delta S}{v} = \frac{79.2m}{0.5 \cdot c} \doteq \boxed{5.28\times10^{-7}s}$$ (ANS)

(17) In a game of RELATIVISTIC CHICKEN,[†] you and I fly our starships toward each other. I think my space racer is 30.0m long. How long do you think it is if we go 0.90c?

I am at rest with respect to my space racer, so I measure its PROPER LENGTH. To get the observed length seen by you, I need γ:

$$\gamma = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} = \left[1 - (0.9)^2\right]^{-1/2} = 2.29$$

$$\text{so} \quad \Delta L = \frac{\Delta \lambda}{\gamma} = \frac{30.0\text{m}}{2.29} = \boxed{13.1\text{m}} \text{ (ANS)}$$

† For "NON-RELATIVISTIC CHICKEN" (with tractors) see the classic 1984 movie *FOOTLOOSE*.

(34) An electron has a momentum 2.4×10^{-22} kg m/s. What is its speed?

Relativistic momentum is: $p = m\gamma v$. There is a v hiding in γ, so it is the only unknown here (m I can look up).

$$p = \gamma m v = \frac{mv}{\sqrt{1-(v/c)^2}}$$

SQUARE BOTH SIDES

$$p^2 = \frac{m^2 v^2}{1-\frac{v^2}{c^2}} \longrightarrow p^2\left(1-\frac{v^2}{c^2}\right) = m^2 v^2$$

$$p^2 - p^2\frac{v^2}{c^2} = m^2 v^2 \longrightarrow m^2 v^2 + \frac{p^2 v^2}{c^2} = p^2$$

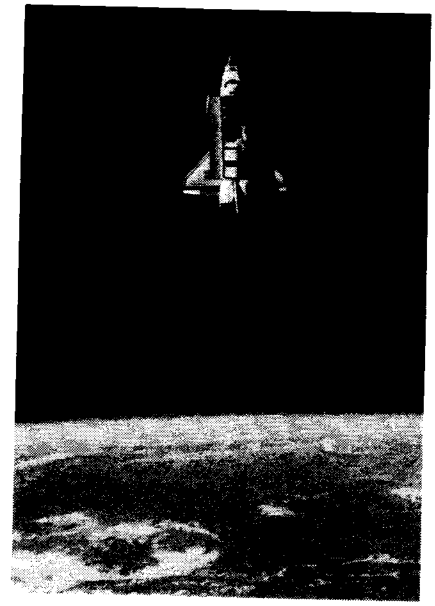
$$v^2\left(m^2 + \frac{p^2}{c^2}\right) = p^2 \rightarrow v^2 = \frac{p^2}{m^2 + p^2/c^2}$$

$$\frac{v^2}{c^2} = \frac{p^2}{c^2\left(m^2 + p^2/c^2\right)} \longrightarrow \boxed{\left(\frac{v}{c}\right)^2 = \frac{p^2}{(m^2c^2 + p^2)}}$$

NUMERICS:

$$\frac{v}{c} = \sqrt{\frac{p^2}{m^2c^2 + p^2}} = \left[\frac{(2.4 \times 10^{-22}\,\mathrm{kg\,m/s})^2}{(9.11 \times 10^{-31}\,\mathrm{kg} \cdot 3.0 \times 10^{8}\,\mathrm{m/s})^2 + (2.4 \times 10^{-22}\,\mathrm{kg\,m/s})^2}\right]^{1/2}$$

or// $\boxed{v = 0.66\,c}$ (ANS)



(35) The space shuttle has a mass of 1×10^5 kg and travels at a speed of about 8×10^3 m/s. What is the shuttle's momentum? What is the % diff between relativistic and classical momenta?

The shuttle's classic momentum is:

$$p = mv = (1\times10^5\,kg)(8\times10^3\,m/s) = 8.0\times10^8\,kg\,m/s$$

Relativistic momentum is: $p = \gamma m v$. In this case:

$$v = 8\times10^3\,m/s = 2.67\times10^{-5}c$$

so/

$$v \ll c \quad \text{and} \quad \gamma \simeq 1 + \tfrac{1}{2}\left(\tfrac{v}{c}\right)^2 \simeq 1 + 3.56\times10^{-10}$$

For all intents and puposes $p_{SR} \simeq p_c \doteq \boxed{8.0\times10^8\,kg\,m/s}$

The exact relativistic momentum:

$$p_{SR} = mv\left(1 + \tfrac{1}{2}\left(\tfrac{v}{c}\right)^2\right) = (1\times10^5\,kg)(8\times10^3\,m/s)\left[1 + 3.56\times10^{-10}\right]$$

$$= 8.0\times10^8\,kg\,m/s + 0.28\,kg\,m/s$$

$$\%D = \frac{p_{SR} - p_c}{p_{SR}}\times100 = \frac{0.28\,kg\,m/s}{8.0\times10^8\,\frac{kg\,m}{s} + 0.28\,\frac{kg\,m}{s}}\times100 \doteq \boxed{3.56\times10^{-8}\%}$$

(Ans)

(38) How much energy is released if 1.0g of fuel is converted?

This is a simple application of $E=mc^2$:

$$E=(1.0\times10^{-3}\,kg)(3.0\times10^{8}\,m/s)^2 = \boxed{9.0\times10^{13}\,J}$$ (ANS)

(40) The solar constant at the surface of the Earth's atmosphere is 1.4 kW/m². (a) How much mass does Sun burn/day? (b) What % of solar mass is this?

Intensity is: $I=\frac{P}{A}=\frac{(E/t)}{A}=\frac{E}{\Delta t\cdot 4\pi r^2}=\frac{mc^2}{\Delta t\cdot 4\pi r^2}$

ASSUME A SPHERE

or//

$$\boxed{m=\frac{I\cdot\Delta t\cdot 4\pi r^2}{c^2}}$$

Here, the area $4\pi r^2$ is the area the sun illuminates, a sphere with the radius of the Earth's orbit.

so//
$$m=\frac{(1.4\times10^{3}\,W/m^2)(86400s)(4\pi)(1.5\times10^{11}m)^2}{(3\times10^{8}\,m/s)^2}=\boxed{3.8\times10^{14}\,kg}$$

(ANS) !

Big number! Compared to the total solar mass:

$$\%D=\frac{m_{SUN}-m}{m_{SUN}}\times100=\left(\frac{1.99\times10^{30}kg-3.8\times10^{14}kg}{1.99\times10^{30}\,kg}\right)\times100$$

$$=1-\frac{3.8\times10^{14}kg}{1.99\times10^{30}kg}=(1-\underbrace{1.91\times10^{-16}}_{\text{percent buned.}})\,\%$$