Φ2020 - MANIC MONDAY #6
SPECIAL RELATIVITY

① (a) We are going to consider the LIFETIME of the particle
(b) We will use: $\Delta t = \gamma \Delta\tau$
(c) The particle (kaon) measures PROPER LIFETIME $\Delta\tau$ (the particle is born and decays at the same location as its clock)
(d) YOU and I measure OBSERVED TIME Δt (the particle moves—it is born in one place and dies in another)

$$V = 0.997c \Rightarrow \gamma = \left[1-\left(\tfrac{v}{c}\right)^2\right]^{-1/2} = \left[1-(0.997)^2\right]^{-1/2} = 12.92$$

$$\left.\begin{array}{l} \text{so// } \gamma = 12.92 \\ \Delta\tau = 1.24\times10^{-8}\,s \end{array}\right\} \Delta t = \gamma\Delta\tau = (12.92)(1.24\times10^{-8}s) \doteq \boxed{1.60\times10^{-7}s}$$

(ANS)

②(a) We are measuring a distance the particle travels.
(b) We can use: $V = \Delta S/\Delta T$ where v is known and $\Delta T = \Delta t$.
(c) We are measuring a distance between points in our lab; they are at rest with respect to us so we measure PROPER LENGTH
(d) The K^+ is moving with respect to our lab markers, so sees an OBSERVED LENGTH ΔL.

$$V = \frac{\Delta S}{\Delta T} \Rightarrow \Delta S = V\cdot\Delta T = (0.997c)(1.60\times10^{-7}s) \doteq \boxed{47.9m}$$

We measure the accelerator to be 30m long, so the K^+ easily make it out of the accelerator and into our patient.

$\longrightarrow$

③ (a) The time to traverse the length of the accelerator.

(b) $v = \Delta s / \Delta T$ where v is known and Δs = length of accelerator

(c) The particle (K^+) measures PROPER TIME since the events ((crossing ends of accelerator) occur at the same place for the particle.

(d) We see the events at different places, so see OBSERVED TIME.

$$v = \frac{\Delta s}{\Delta T} \Rightarrow \Delta T = \Delta t = \frac{\Delta s}{v} = \frac{30\,m}{0.997c} \doteq \boxed{1.00 \times 10^{-7} s = \Delta t} \text{ (ANS)}$$

④ (a) The length of the accelerator

(b) We will use: $\Delta L = \Delta\lambda / \gamma$ [Remember: $\gamma = 12.92$]

(c) The accelerator is at rest in the lab, so we measure the proper length $\Delta\lambda = 30m$.

(d) The Kaon is moving with respect to the accelerator, so sees an OBSERVED LENGTH ΔL

The kaon sees a contracted length:

$$\Delta L = \frac{\Delta\lambda}{\gamma} = \frac{30\,m}{12.92} \doteq \boxed{2.32\,m} \text{ (ANS)}$$

→

⑤ (a) The time to traverse the length of the accelerator.

(b) $V = \Delta S / \Delta T$ where v is known and ΔS = length of accelerator according to the particle (see #4)

(c) The particle measures proper time since events occur at same place

(d) We see events at different places and measure OBSERVED TIME.

$$V = \frac{\Delta S}{\Delta T} \rightarrow \Delta T = \Delta \tau = \frac{\Delta S}{V} = \frac{2.32\,m}{0.997c} = \boxed{7.76 \times 10^{-9}\,s} \text{ (ANS)}$$

Note I could have just as easily used time dilation and the result from ③: $\Delta t = \gamma \Delta \tau \rightarrow \Delta \tau = \frac{\Delta t}{\gamma} = \frac{1.0 \times 10^{-7}\,s}{12.92} = 7.7 \times 10^{-9}\,s$.

GPS

① The satellite has a speed $v = 3900\,m/s = 1.3009 \times 10^{-5}\,c$

so//

$$\gamma = \left[1 - (v/c)^2\right]^{-1/2} = \boxed{1.0000}$$

② The binomial approximation is: $\gamma \simeq 1 + \frac{1}{2}\left(\frac{v^2}{c^2}\right)$

so//

$$1 - \gamma = 1 - \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = \boxed{(-)\frac{1}{2}\frac{v^2}{c^2}}$$

→

③ The satellite measures proper time $\Delta\tau$ so:

$$\Delta t = \gamma \Delta\tau$$

Since v is small, apply $\gamma \simeq 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2$ so,

$$\Delta t = \left(1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right)\Delta\tau = \Delta\tau + \frac{1}{2}\Delta\tau\left(\frac{v}{c}\right)^2$$

or,

$$\underbrace{\Delta t - \Delta\tau}_{\text{Difference Between clocks}} = \underbrace{\frac{1}{2}\Delta\tau\left(\frac{v}{c}\right)^2}_{\text{Things I know.}}$$

so,

$$\Delta t - \Delta\tau = \frac{1}{2}(86400\,s)\left(\frac{3900\,m/s}{3.00\times10^{8}\,m/s}\right)^2 = \boxed{7.31\times10^{-6}\,s}$$

So without SR, GPS clocks will be slow by 7.3 μs!

④ We can show positional error from this by: $V = \Delta S / \Delta T$ where V = speed of light and ΔT = time error above so,

$$V = \frac{\Delta S}{\Delta T} \rightarrow \Delta S = V\Delta T = \left(3.0\times10^{8}\,\frac{m}{s}\right)\left(7.31\times10^{-6}\,s\right) = \boxed{2192\,m}$$

Without SR, GPS clocks will slow, and produce 2 km position errors in just ONE DAY!