

Φ 2020 - MANIC MONDAY #6
SPECIAL RELATIVITY

- ① (a) We are going to consider the LIFETIME of the particle
(b) We will use: $\Delta t = \gamma \Delta \tau$
(c) The particle (kaon) measures PROPER LIFETIME $\Delta \tau$ (the particle is born and decays at the same location as its clock)
(d) You and I measure OBSERVED TIME Δt (the particle moves - it is born in one place and dies in another)

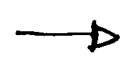
$$v = 0.997c \Rightarrow \gamma = \left[1 - \left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} = \left[1 - (0.997)^2\right]^{-\frac{1}{2}} = 12.92$$

$$\left. \begin{array}{l} \text{so } \gamma = 12.92 \\ \Delta \tau = 1.24 \times 10^{-8} \text{ s} \end{array} \right\} \Delta t = \gamma \Delta \tau = (12.92)(1.24 \times 10^{-8} \text{ s}) = \boxed{1.60 \times 10^{-7} \text{ s}} \quad (\text{ANS})$$

- ② (a) We are measuring a distance the particle travels.
(b) We can use: $v = \Delta s / \Delta t$ where v is known and $\Delta t = \Delta t$.
(c) We are measuring a distance between points in our lab; they are at rest with respect to us so we measure PROPER LENGTH
(d) The K^+ is moving with respect to our lab markers, so sees an OBSERVED LENGTH ΔL .

$$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta s = v \cdot \Delta t = (0.997c)(1.60 \times 10^{-7} \text{ s}) = \boxed{47.9 \text{ m}}$$

We measure the accelerator to be 30m long, so the K^+ easily make it out of the accelerator and into our patient.



- ③ (a) The time to traverse the length of the accelerator.
- (b) $v = \Delta s / \Delta t$ where v is known and $\Delta s =$ length of accelerator
- (c) The particle (K^+) measures PROPER TIME since the events (crossing ends of accelerator) occur at the same place for the particle.
- (d) We see the events at different places, so see OBSERVED TIME.

$$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta T = \Delta t = \frac{\Delta s}{v} = \frac{30\text{m}}{0.997c} = \boxed{1.00 \times 10^{-7}\text{s} = \Delta t} \text{ (ANS)}$$

- ④ (a) The length of the accelerator
- (b) We will use: $\Delta L = \Delta \lambda / \gamma$ [Remember: $\gamma = 12.92$]
- (c) The accelerator is at rest in the lab, so we measure the proper length $\Delta \lambda = 30\text{m}$.
- (d) The Kaon is moving with respect to the accelerator, so sees an OBSERVED LENGTH ΔL

The kaon sees a contracted length:

$$\Delta L = \frac{\Delta \lambda}{\gamma} = \frac{30\text{m}}{12.92} = \boxed{2.32\text{m}} \text{ (ANS)}$$



- ⑤ (a) The time to traverse the length of the accelerator.
- (b) $v = \Delta s / \Delta T$ where v is known and $\Delta s =$ length of accelerator according to the particle (see #4)
- (c) The particle measures proper time since events occur at same place
- (d) We see events at different places and measure OBSERVED TIME.

$$v = \frac{\Delta s}{\Delta T} \rightarrow \Delta T = \Delta \tau = \frac{\Delta s}{v} = \frac{2.32 \text{ m}}{0.997c} = \boxed{7.76 \times 10^{-9} \text{ s}} \text{ (ANS)}$$

Note I could have just as easily used time dilation and the result from ③: $\Delta t = \gamma \Delta \tau \rightarrow \Delta \tau = \frac{\Delta t}{\gamma} = \frac{1.0 \times 10^{-7} \text{ s}}{12.92} = 7.7 \times 10^{-9} \text{ s}.$

GPS

① The satellite has a speed $v = 3900 \text{ m/s} = 1.3009 \times 10^{-5} c$

so// $\gamma = [1 - (v/c)^2]^{-1/2} = \boxed{1.0000}$

② The binomial approximation is: $\gamma \approx 1 + \frac{1}{2} (\frac{v^2}{c^2})$

so// $1 - \gamma = 1 - (1 + \frac{1}{2} \frac{v^2}{c^2}) = \boxed{\Rightarrow \frac{1}{2} \frac{v^2}{c^2}}$



③ The satellite measures proper time $\Delta\tau$ so:

$$\Delta t = \gamma \Delta\tau$$

Since v is small, apply $\gamma \approx 1 + \frac{1}{2}(\frac{v}{c})^2$ so,

$$\Delta t = (1 + \frac{1}{2}(\frac{v}{c})^2) \Delta\tau = \Delta\tau + \frac{1}{2} \Delta\tau (\frac{v}{c})^2$$

or,

$$\underbrace{\Delta t - \Delta\tau}_{\substack{\uparrow \text{Difference} \\ \text{Between clocks}}} = \frac{1}{2} \underbrace{\Delta\tau (\frac{v}{c})^2}_{\substack{\uparrow \text{Things I} \\ \text{know.}}}$$

$$\text{so, } \Delta t - \Delta\tau = \frac{1}{2} (86400s) \left(\frac{3900 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2 = \boxed{7.31 \times 10^{-6} \text{ s}}$$

So without SR, GPS clocks will be slow by 7.3 μs !

④ We can show positional error from this by: $\Delta s = v \Delta T$
where v = speed of light and ΔT = time error above
so,

$$v = \frac{\Delta s}{\Delta T} \rightarrow \Delta s = v \Delta T = (3.0 \times 10^8 \frac{\text{m}}{\text{s}}) (7.31 \times 10^{-6} \text{ s}) = \boxed{2192 \text{ m}}$$

Without SR, GPS clocks will slow, and produce 2 km position errors in just ONE DAY!